**2019 May Day MCM**

**Problem B: The optimal board cutting scheme**

A furniture factory in Xuzhou has a batch of the board as shown in Table 1. In the process of furniture processing, it is necessary to use the cutting machine to produce products as required in Table 2. Assumption: the thickness of the board and the width of the slot can be neglected.

Table 1 The size of board

|  |  |  |
| --- | --- | --- |
| Type of board | Length(mm) | Width(mm) |
| S1 | 3000 | 1500 |

Table 2 The size of products and production task

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Type of product | Length(mm) | Width(mm) | Production task(piece) | Profit(Yuan/piece) |
| P1 | 373 | 201 | 774 | 19.9 |
| P2 | 477 | 282 | 2153 | 23.0 |
| P3 | 406 | 229 | 1623 | 21.0 |
| P4 | 311 | 225 | 1614 | 16.0 |

Please design the optimal board cutting scheme of the following problems for the furniture factory.

1. Produce P1 using one board. Please establish a mathematical model so as to give a cutting scheme with the highest utilization ratio of the board (The area of the remaining board is minimum). The results of the optimal scheme are needed to be filled in Table 3.

Table 3 The results of problem 1

|  |  |
| --- | --- |
| Quantity of P1 | Utilization ratio of the board |
|  |  |

1. Produce P1 and P3 using one board. Please establish a mathematical model so as to give the three cutting schemes, which ranks the top three with respect to the board utilization ratio from high to low. The results are needed to be filled in Table 4.

Table 4 The results of problem 2

|  |  |  |  |
| --- | --- | --- | --- |
| Scheme number | Quantity of P1 | Quantity of P3 | Utilization ratio of the board |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

1. Regarding the requirement of completing the production tasks of P1 and P3 in Table 2, please establish a mathematical model so as to give the cutting scheme with the highest total utilization ratio of the board. The results are needed to be filled in Table 5.

Table 5 The results of problem 3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Quantity of S1 | Quantity of P1 | Quantity of P3 | Utilization ratio of the board | Note |
|  |  |  |  | Each board has the same cutting scheme |
| …… |  |  |  | Ditto  add the row if necessary |
| Total quantity：  \_\_\_\_\_\_\_\_ | 774 | 1623 | Total utilization ratio of all the boards:  \_\_\_\_\_\_\_\_ | Total utilization ratio of all the boards= |

1. Regarding the requirement of completing the production tasks of P1, P2, P3 and P4 in Table 2, please establish a mathematical model so as to give the cutting scheme with the highest total utilization ratio of all the boards. The results are needed to be filled in Table 6.

Table 6 The results of problem 4

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Quantity of S1 | Quantity of P1 | Quantity of P2 | Quantity of P3 | Quantity of P4 | Utilization ratio of the board | Note |
|  |  |  |  |  |  | Each board has the same cutting scheme |
| …… |  |  |  |  |  | Ditto  add the row if necessary |
| Total quantity：  \_\_\_\_\_\_\_\_ | 774 | 2153 | 1623 | 1614 | Total utilization ratio of all the boards:  \_\_\_\_\_\_\_\_ | Total utilization ratio of all the boards= |

1. Assumption: there are totally 100 boards (S1) in the furniture factory and do not need to consider the production tasks of P1, P2, P3 and P4. Please establish a mathematical model so as to give the cutting scheme with the highest total profit based on the profit of each product in Table 2. The results are needed to be filled in Table 7.

Table 7 The results of problem 5

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Quantity of S1 | Quantity of P1 | Quantity of P2 | Quantity of P3 | Quantity of P4 | Profit | Utilization ratio of the board | Note |
|  |  |  |  |  |  |  | Each board has the same cutting scheme |
| …… |  |  |  |  |  |  | Ditto  add this row if necessary |
| Total quantity of S1: 100 |  |  |  |  | Total profit:  \_\_\_\_\_\_\_\_ | Total utilization ratio of all the boards:  \_\_\_\_\_\_\_\_ | Total utilization ratio of all the boards= |